J Math Chem (2011) 49:325–327
DOI 10.1007/s10910-010-9759-2

LETTER TO THE EDITOR

On the zeroth-order general Randić index

Minjie Zhang · Shuchao Li

Published online: 28 October 2010

In [1], the authors obtained two lemmas as follows.

(•) (Lemma 3.8 of [1]) *Let* $\alpha > 1$ *or,* $\alpha < 0$*, and* G *be a graph in* $\mathscr{U}^k_{2m,m}$ *such that* $R^0_\alpha(G)$ *is as large as possible, then for each* $v_i \in S$*, we have* $T(v_i) \cong T^0\left(n_i, \frac{n_i}{2}\right)$ *or,* $T(v_i) \cong T^0\left(n_i, \frac{n_i-1}{2}\right)$*. Moreover, if* $T(v_i) \cong T^0\left(n_i, \frac{n_i}{2}\right)$*, then* $d(v_i) - 2 = \Delta\left(T^0\left(n_i, \frac{n_i}{2}\right)\right)$*; if* $T(v_i) \cong T^0\left(n_i, \frac{n_i-1}{2}\right)$*, then* v_i *is one pendent vertex of* $T^0\left(n_i, \frac{n_i-1}{2}\right)$ *which is adjacent to the maximum-degree vertex of* $T^0\left(n_i, \frac{n_i-1}{2}\right)$*.*

(⋄) (Lemma 3.10 of [1]) *Let* $0 < \alpha < 1$ *and* G *be a graph in* $\mathscr{U}^k_{2m,m}$ *such that* $R^0_\alpha(G)$ *is as small as possible, then for each* $v_i \in S$*, we have* $T(v_i) \cong T^0\left(n_i, \frac{n_i}{2}\right)$ *or,* $T(v_i) \cong T^0\left(n_i, \frac{n_i-1}{2}\right)$*. Moreover, if* $T(v_i) \cong T^0\left(n_i, \frac{n_i}{2}\right)$*, then* $d(v_i) - 2 = \Delta\left(T^0\left(n_i, \frac{n_i}{2}\right)\right)$*;* if $T(v_i) \cong T^0\left(n_i, \frac{n_i-1}{2}\right)$, then v_i *is one pendent* vertex of $T^0\left(n_i, \frac{n_i-1}{2}\right)$ *which is adjacent to the maximum-degree vertex of* $T^0\left(n_i, \frac{n_i-1}{2}\right)$.

Financially supported by self-determined research funds of CCNU (CCNU09Y01005, CCNU09Y01018) from the colleges' basic research and operation of MOE and the National Science Foundation of China (Grant No. 11071096).

M. Zhang · S. Li (✉)
Faculty of Mathematics and Statistics, Central China Normal University, 430079 Wuhan,
People's Republic of China
e-mail: lscmath@mail.ccnu.edu.cn

M. Zhang
Faculty of Mathematics and Physics, Huangshi Institute of Technology, 435003 Huangshi,
People's Republic of China
e-mail: zmj1982@21cn.com

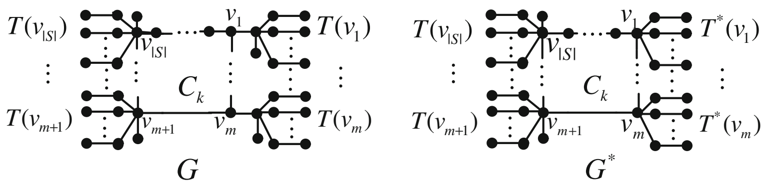

Fig. 1 Graphs G and G^*

In fact, without loss of generality, let $S = \{v_1, v_2, \ldots, v_m, v_{m+1}, \ldots, v_{|S|}\}$ such that n_1 (respectively, $n_2, n_3, \ldots, n_m$) is odd and n_{m+1} (respectively, $n_{m+2}, n_{m+3}, \ldots, n_{|S|}$) is even. If the claim in ($\bullet$) (respectively, ($\diamond$)) is true, then graph G depicted in Fig. 1 is the conjugated unicyclic graph with maximal (respectively, minimal) R^0_α-value when $\alpha > 1$ or, $\alpha < 0$ (respectively, $0 \le \alpha < 1$), where $T(v_i) \cong T^0(n_i, \frac{n_i-1}{2})$, $i = 1, 2, \ldots, m$ and $T(v_j) \cong T^0(n_j, \frac{n_j}{2})$, $j = m+1, m+2, \ldots, |S|$. Let G^* be the conjugated n-vertex unicyclic graph obtained from G by replacing $T(v_i)$ by $T^*(v_i)$, $i = 1, 2, \ldots, m$, where $T^*(v_i) \cong T^0(n_i, \frac{n_i+1}{2})$; see Fig. 1. Hence, $\Delta(T^*(v_i)) = \Delta(T(v_i)) - 1$, $i = 1, 2, \ldots, m$. By the definition of zeroth-order general Randić index, we have

$$\begin{aligned} R^0_\alpha(G) - R^0_\alpha(G^*) &= \sum_{i=1}^{m} \left[3^\alpha + (\Delta(T(v_i)))^\alpha\right] - \sum_{i=1}^{m} \left[2^\alpha + (\Delta(T^*(v_i)))^\alpha\right] \\ &= \sum_{i=1}^{m} \left[3^\alpha - 2^\alpha + (\Delta(T(v_i)))^\alpha - (\Delta(T^*(v_i)))^\alpha\right] \\ &= \sum_{i=1}^{m} \left[\alpha \left(\xi_i^{\alpha-1} - \eta_i^{\alpha-1}\right)\right], \end{aligned}$$

where $2 < \xi_i < 3 \le \Delta(T^*(v_i)) < \eta_i < \Delta(T(v_i))$, $i = 1, 2, \ldots, m$. Therefore, when $\alpha > 1$ or, $\alpha < 0$, we obtain $R^0_\alpha(G) < R^0_\alpha(G^*)$, a contradiction; when $0 < \alpha < 1$, we have $R^0_\alpha(G) > R^0_\alpha(G^*)$, a contradiction. Therefore, both ($\bullet$) and ($\diamond$) are false.

Based on ($\bullet$) and ($\diamond$), the author obtained the following three main results in [1] (Thereby, all of them are not correct).

($\lhd$) (Theorem 3.14 (i) and (iii) of [1]). *Suppose $G \in \mathscr{U}^k_{2m,m}$, and $\alpha > 2$, we have the following:*

(i) If $2m = k + 2$, then $R^0_\alpha(G) \le 2 + (k-2)2^\alpha + 3^\alpha$ with equality holding if and only if $G \ncong (C_k, v_i) \bowtie (P_3, v_i)$.

(iii) If $2m \ge k + 3$ and k is even, then $R^0_\alpha(G) \le (m - \frac{k}{2}) + (m + \frac{k}{2} - 2)2^\alpha + 3^\alpha + (m + 1 - \frac{k}{2})^\alpha$ with equality if and only if $G \cong (C_k, v_i) \bowtie (T^0(2m - k + 1, \frac{2m-k}{2}), v_i)$. Moreover, $d(v_i) = 3$ and u is the maximum-degree vertex of $(T^0(2m - k + 1, \frac{2m-k}{2}), v_i)$ where $u = N(v_i) - \{v_{i-1}, v_{i+1}\}$.

(▷) (Theorem 3.15 (ii) of [1]). *Suppose* $G \in \mathscr{U}^k_{2m,m}$, *and* $\alpha > 1$ *or* $\alpha < 0$. *If* $|V(T(v_i))| \geq 3$ *for each* $v_i \in S$, *then*

(ii) *If* k *is even, then* $R^0_\alpha(G) \leq (m - \frac{k}{2}) + (m + \frac{k}{2} - 2)2^\alpha + 3^\alpha + (m + 1 - \frac{k}{2})^\alpha$ *with equality if and only if* $G \cong (C_k, v_i) \bowtie (T^0(2m - k + 1, \frac{2m-k}{2}), v_i)$. Moreover, $d(v_i) = 3$ *and* u *is the maximum-degree vertex of* $(T^0(2m - k + 1, \frac{2m-k}{2}), v_i)$ *where* $u = N(v_i) - \{v_{i-1}, v_{i+1}\}$.

(∘) (Theorem 3.16 (ii) of [1]). *Suppose* G *is a graph in* $\mathscr{U}^k_{2m,m}$ *and* $0 < \alpha < 1$. *If* $|V(T(v_i))| \geq 3$ *for each* $v_i \in S$, *then*

(ii) *when* k *is even,* $R^0_\alpha(G) \geq \left(m - \frac{k}{2}\right) + \left(m + \frac{k}{2} - 2\right) 2^\alpha + 3^\alpha + \left(m + 1 - \frac{k}{2}\right)^\alpha$ *with equality if and only if* $G \cong (C_k, v_i) \bowtie \left(T^0\left(2m - k + 1, \frac{2m-k}{2}\right), v_i\right)$. *Moreover,* $d(v_i) = 3$ *and* u *is the maximum-degree vertex of* $(T^0\left(2m - k + 1, \frac{2m-k}{2}\right), v_i)$ *where* $u = N(v_i) - \{v_{i-1}, v_{i+1}\}$.

Reference

1. H. Hua, M. Wang, H. Wang, On zeroth-order general Randić index of conjugated unicyclic graphs. J. Math. Chem. **43**(2), 737–748 (2007)